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[This question paper contains 12 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4748 E

Unique Paper Code : 32357609

Name of the Paper : DSE-3 : Bio-Mathematics

Name of the Course : B.Sc. (H) Mathematics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the six questions are compulsory.
3. Attempt any two parts from each question.
4. Use of Scientific Calculator is allowed.

1. (a) A doctor has to prescribe medicine to the patient.

The medicine raises the blood plasma concentration of an average adult by 20 mg l^{-1} and takes 6 h to decay in the blood plasma. The maximum

P.T.O.

permissible limit of concentration of drug in the body is 40 mg l^{-1} . What time gap he will ensure for maintaining a safe decomposition of drug. Find out the minimum concentration if doses are given at this time interval? The concentration of another drug, decreases by 40% in 20 h. Find how long will it take for this drug to fall to 5% of its initial value. (6)

(b) Observation on animal tumours indicate that their sizes obey the Gompertz growth law

$$\frac{ds}{dt} = ks \ln\left(\frac{S}{s}\right) \text{ rather than the logistic law. Here } k$$

and S are positive constants. By putting $y = \ln(s)$,

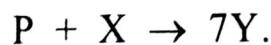
$$\text{prove that } s(t) = S e^{-Ae^{-kt}}, \text{ where } A = \ln\left(\frac{S}{s_0}\right), s_0$$

being the size at $t = 0$. Discuss the model describing drug concentration and residual concentration at any time t , in which drug decays

according to equation $\frac{dc}{dt} = \frac{-c(t)}{\tau}$. When dose is administered regularly at time $t = 0, t_0, 2t_0,$

$3t_0, \dots$ with assumption that each dose raises the drug concentration by fixed amount C_0 . Find the maximum possible concentration and residue as n increases. (6)

- (c) Consider the following chemical reaction, with the rate constant as q :



If the reactant P is held at a constant concentration p , derive a system of equations for the

P.T.O.

concentrations of X and Y. Suppose the initial concentrations of X and Y are X_0 and Y_0 respectively. Solve the system of equations to obtain $X(t)$ and $Y(t)$. (6)

(d) Show that Zeeman's heartbeat equations have a unique resting state

$$\dot{x} = x_a, \quad \dot{b} = -(x_a^3 + a x_a).$$

Then derive the single differential equation satisfied by muscle fibre of length x . (6)

2. (a) Discuss the nature of fixed point and give equation of trajectories for the given system

$$\begin{aligned} \dot{x} &= 6x + 12y \\ \dot{y} &= 3x + y \end{aligned} \quad (6\frac{1}{2})$$

(b) (i) Discuss the two equilibrium states during the heart beat cycle and the role of pacemaker in the heartbeat cycle.

(ii) Discuss the threshold level and firing of axon in nerve impulse transmission. (6½)

(c) Examine the possibility of periodic solutions of

$$c\ddot{x} + (2 + 3ax + 4bx^2)x = 0$$

Where a , b and c are constants, c being positive. (6½)

(d) Describe the epidemic model and show that population return to equilibrium after the small departure from the equilibrium. (6½)

3. (a) Consider the system

$$\frac{du}{dt} = u(1-u)(u-a) - w$$

$$\frac{dw}{dt} = bu - \gamma w.$$

Where $0 < a < 1$, $b > 0$, $\gamma \geq 0$.

Linearise the above system about $(0,0)$. Further assuming that $u = \alpha e^{\lambda t}$, $w = \beta e^{\gamma t}$ be solutions of the linearised system about $(0,0)$, show that the rest state $(0,0)$ is locally stable. (6)

(b) Sketch the trajectories of the following system

$$\dot{x} = y$$

$$\dot{y} = \frac{1}{2}(1 - x^2) \quad (6)$$

(c) Define

(i) Bifurcation

(ii) Bifurcation Point

Make the sketches for Pitchfork bifurcation,

Saddle-node bifurcation and Hopf bifurcation.

(6)

(d) For the iteration scheme $x_{n+1} = \mu x_n(1 - x_n)$, $n \geq 1$,

$$x_0 = \lim_{n \rightarrow \infty} x_n$$

Show that there are bifurcations at $\mu = 1$ and $\mu = 3$. (6)

4. (a) Find the constraints on a , b and λ assuming it has a unique rest state, taking the solutions to the travelling wave equations in the form $u = \phi(x + ct)$, $w = \psi(x + ct)$ of the following

system of equations $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u(1-u)(u-a) - w$,

$$\frac{\partial w}{\partial t} = bu - \gamma w. \quad (6\frac{1}{2})$$

(b) What is Flopf bifurcation? Show that Hopf

bifurcation holds for the following system

$$\dot{x} = -y + x(\mu - x^2 - y^2)$$

$$\dot{y} = x + y(\mu - x^2 - y^2) \quad (6\frac{1}{2})$$

(c) Provide a full phase plane analysis for the mathematical model of heart beat equations given by

$$\varepsilon \frac{dx}{dt} = -(x^3 - Tx + b), \quad T > 0$$

$$\frac{db}{dt} = x - x_0.$$

Where x is the muscle fibre length, b is the chemical control, $\varepsilon > 0$ and (x_0, b_0) is a rest state. (6 $\frac{1}{2}$)

(d) Show that the following system has limit cycle.

$$\frac{du}{dt} = u(1-u)(u-a) - w + I(t),$$

$$\frac{dw}{dt} = bu - \gamma w, \quad 0 < a < 1, \quad b > 0, \quad \gamma \geq 0. \quad (6\frac{1}{2})$$

5. (a) Write down the steps of Neighbor Joining Algorithm. From the given distance table of four taxa S_1, S_2, S_3 and S_4 , compute R_1, R_2, R_3, R_4 and then form a table of values for $M(S_i, S_j)$ for $1 \leq i \neq j \leq 4$.

	S_1	S_2	S_3	S_4
S_1		1.2	0.9	1.7
S_2			1.1	1.9
S_3				1.6

(6½)

- (b) If D and d denote the alleles for tall and dwarf plant and if W and w denote the alleles for round

and wrinkled seed, then create a Punnett square for a $DdWw \times ddWw$ cross pea plant and compute the probability of a tall plant with wrinkled seeds. (6½)

(c) Derive the formula for the Jukes-Cantor distance

(d_{JC}) given that all the diagonal entries of Jukes

Cantor matrix M^t are $\frac{1}{4} + \frac{3}{4} \left(1 - \frac{4}{3}\alpha\right)^t$, where α is

the mutation rate. Compute the Jukes-Cantor

distance $d_{JC}(S_0, S_1)$ to 4 decimal digits, from the

following 40 base table :

(6½)

$S_i \backslash S_0$	A	G	C	T
A	7	0	1	1
G	1	9	2	0
C	0	2	7	2
T	1	0	1	6

- (d) Describe when two trees are considered to be topologically similar. Draw all topologically distinct un-rooted bifurcation trees that could describe the relationship between 3 taxa and 4 taxa. (6½)
6. (a) Define phylogenetic tree, bifurcating tree and unrooted tree with examples of each. (6)
- (b) Explain Kimura 2-parameter and 3-parameter models along with their corresponding distance formulas. Write the expression of the log-det distance between S_0 and S_1 . (6)
- (c) In mice, an allele A for agouti- or gray-brown grizzled fur is dominant over the allele a , which determines a non-agouti color. If an $Aa \times Aa$ cross produces 4 offsprings, then compute the probabilities that :
- (i) No offspring have agouti fur.
- (ii) Exactly 3 of 4 offspring have agouti fur. (6)

- (d) From the given distance table of four sequences S_1 , S_2 , S_3 and S_4 of DNA, construct a rooted tree showing the relationship between S_1 , S_2 , S_3 and S_4 by UPGMA

	S_1	S_2	S_3	S_4
S_1		0.45	0.27	0.53
S_2			0.40	0.50
S_3				0.62

(6)